String-Steilkurs Teil II: 2009
Flux Compactifications

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von Marco Schreck.

Dieser Mitschrieb erhebt keinen Anspruch auf Vollständigkeit und Korrektheit.
Kommentare, Vorschläge und konstruktive Kritik bitte an Marco.Schreck@gmx.de.

Contents

Chapter 1

Flux compactifications

1.1 Motivation

Superstring theory is only well-defined in $D = 10$. However, we do not live in ten dimensions, but in four. The **basic idea** is that one considers strings in $\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6$, whereas $\mathcal{M}_4$ is the visible part and $\mathcal{M}_6$ is the internal compact space, which is not directly observed. From mathematics we know that there is **discrete information** that characterizes such a space, in other words some topological information (for example intersection numbers) specify $\mathcal{M}_6$. **However**, there are also continuous deformations of this internal space, for example shape and size of the internal manifold. These deformations appear as **scalar fields** in $D = 4$ effective theory. In general, these have no potential and are massless. That is a problem, because we have not observed massless (or very light) scalar fields. This is known as the **moduli problem**. Now, so-called "fluxes" come into the game. First of all, they are some discrete quanta or information, which induce a potential and and possibly stabilize this moduli, which means that it generates a potential so that the scalar fields become massive. There are obvious applications at this point:

- Can the vacuo be used for breaking SUSY?
- Can one use fluxes in cosmology in these potentials?
- Fluxes can introduce chiral fields by appropriate mechanisms.

Now to some review articles:

- T. Denef: 0803.1194 (F-theory, flux vacua, statistics)
- Douglas/Kachu: 0610102 (fluxes in IIB/IIA string theory plus statistics)
- Grimm: 0507153 (effective actions plus fluxes)

The outline of the lecture is:

1.) Effective actions in $D = 4$

2.) Fluxes in string compactification

3.) Fluxes in F-theory

1.2 Effective theories from string theory

1.2.1 $D = 10$ Supergravity

At low energies ($R \gg l_s = \sqrt{\alpha'}$) we do not need to consider the full string theory, but we can start with a low-energy approximation. From a string theory point of view these are given by the massless ten-dimensional modes. Type IIA and Type IIB will be the corresponding supergravity theories.

Bosonic fields

- metric tensor field $g_{\mu\nu}$: effective action for this theory is simply the Einstein-Hilbert action

$$S_{\mathrm{EH}} = \frac{1}{2\kappa^2} \int\limits_{\mathcal{M}_D} R * 1 \,, \quad *1 = \mathrm{d}^D x \sqrt{\pm \det(g_{\mu\nu})} \,, \tag{1.1}$$

  where $2\kappa^2 = 16\pi G$.

- scalar fields such as the dilaton ϕ with a kinetic term of the form

$$\int\limits_{\mathcal{M}_D} \mathrm{d}\phi \wedge * \mathrm{d}\phi = \int \mathrm{d}^D x \, \partial_\mu \phi \partial^\mu \phi \, \sqrt{\pm \det(g_{\mu\nu})} \,. \tag{1.2}$$

- antisymmetric tensor fields: for example the one-form $A_1 \sim A_\mu$ or, in general, a p-form

$$A_p = \frac{1}{p!} A_{M_1 \dots M_p} \, \mathrm{d}x^1 \wedge \ldots \wedge \mathrm{d}x^p \,. \tag{1.3}$$

  The one-form (vector) satisfies the Maxwell equation $\mathrm{d}F_2 = 0$, $d * F_2 = 0$. Generally

$$\boxed{dF_{p+1} = 0 \,, \quad d * F_{p+1} = 0 \,, \quad (*F_p)_{M_1 \dots M_{D-p}} = \frac{\sqrt{\pm \det(g_{\mu\nu})}}{p!} \varepsilon_{M_1 \dots M_{D-p} N_1 \dots N_p} F^{N_1 \dots N_p} \,.} \tag{1.4}$$

  We will call the first equation the Bianchi identity and the second one the equation of motion. The Bianchi equation is fulfilled automatically, since $F_{p+1} = dA_p$. The equations of motion follows from the action

$$\boxed{S_{\mathrm{form}} = \int F_{p+1} \wedge * F_{p+1} \,.} \tag{1.5}$$

  Anyway

$$F_{p+1} = * dA^{\mathrm{Dual}}_{D-p-2} = * F^{\mathrm{Dual}}_{D-p-2} \,, \tag{1.6}$$

  which is the electro-magnetic duality. Hence, one can decide, with which fields to work.

Fermionic fields

If we can check that our theory is supersymmetric, then the fermionic fields can be determined from the bosonic field content.

M-theory

The low energy effective action is $D = 11$ supergravity ($\mathcal{N} = 1$, 32 supercharges), which we can write down explicitly. Let us first start with the field content of the theory:

- graviton $G_{\mu\nu}$ with 44 degrees of freedom
- 3-form $A_3 = A_{MNP}$ with 84 degrees of freedom
- gravitino ψ_M with 128 degrees of freedom

Having the fields one can write down the bosonic action:

$$S^{(11)} = -\frac{1}{\kappa_{11}^2} \int \left(\frac{1}{2} R * 1 + \frac{1}{4} F_4 \wedge * F_4 + \frac{1}{12} A_3 \wedge F_4 \wedge F_4 \right) \,, \tag{1.7}$$

with the gravitational coupling $2\kappa_{11}^2 = 1/(2\pi)(2\pi l_{\mathrm{Planck}})^9$. The last term in this action is topological, it has no metric in it. It is some kind of Chern-Simons term.

$D = 10$ Type IIA supergravity

This is the low energy limit of Type IIA string theory. It can be obtained from the $D = 11$ supergravity theory by compactification on a circle $R = \sqrt{\alpha'} g_s$, where g_s is the string coupling.

- bosonic fields: g_{IJ}, dilaton ϕ ($\exp(\langle\psi\rangle) = g_s$), NS-NS two-form B_2, R-R one-forms C_1, C_3

The bosonic action is from the shape

$$S_{\mathrm{IIA}}^{\mathrm{string}} = \int \exp(-2\phi)\left(\frac{1}{2}R * 1 - 2\mathrm{d}\phi \wedge *\mathrm{d}\phi + \frac{1}{4}H_3 \wedge *H_3\right) - \int \frac{1}{4}\left(F_2 \wedge *F_2 + \frac{1}{4}F_4 \wedge *F_4\right) + \mathcal{L}_{\mathrm{topological}}\,, \tag{1.8}$$

with $H_3 = dB_2$, $F_2 = dC_1$, and $F_4 = dC_3 - C_1 \wedge H_3$. One can rescale the metric as follows: $g_{IJ}^{\mathrm{string}} = \exp(\phi/2) g_{IJ}^E$. There is a massive IIA supergravity: $F_{10} = dC_9$, $*F_{10} = \mathrm{const.}$ with m being Romans mass. Romans mass is the first flux, which we encounter. This new action has a mass term of the form $-1/2\exp(5/2\phi)m^2 * 1$.

$D = 10$ Type IIB supergravity

The bosonic fields in the NS-NS sector are the same as in IIA. In the R-R sector are not odd forms any more, but even forms C_0, C_2, C_4, C_6, and C_8, whereas C_0 is related to C_8 and C_2 is related to C_6. C_4 has a self-dual field strength F_5 ($F_5 = *F_5$, which has to be imposed at the level of the equations of motion). Bosonic "action":

$$S_{\mathrm{IIB}}^{\mathrm{Einstein}} = -\frac{1}{2}\int \left(\frac{1}{2}R * 1 + \frac{1}{4}\mathrm{d}\phi \wedge *\mathrm{d}\phi + \frac{1}{4}\exp(-\phi)H_3 \wedge *H_3 + \frac{1}{4}\exp(2\phi)F_1 \wedge *F_1 + \frac{1}{4}\exp(\phi)F_3 \wedge *F_3 + \frac{1}{8}F_5 \wedge *F_5 + \frac{1}{4}C_4 \wedge H_3 \wedge F_3\right)\,, \tag{1.9}$$

with $H_3 = dB_2$, $F_1 = dC_0$, $F_{q+1} = dC_q - C_{q-2} \wedge H_3$.
Is there a higher-dimensional theory for IIB string theory/supergravity? F-theory There is a new invariance of the action, which has not been there at the Type IIA case.

SL(2,$\mathbb{Z}$) invariance and first remarks on F-theory

$$\tau = C_0 + \mathrm{i}\exp(-\phi)\,, \quad G_{(3)} = F_3 - \tau H_3\,. \tag{1.10}$$

We can write down the action in a very elegant form:

$$S_{\mathrm{IIB}}^{\mathrm{Einstein}} = -\int \left(\frac{1}{2}R * 1 + \frac{1}{4}\frac{\mathrm{d}\tau \wedge *d\overline{\tau}}{(\mathrm{Im}\tau)^2} + \frac{1}{4}\frac{G_{(3)} \wedge *\overline{G}_{(3)}}{4\mathrm{Im}\tau} - \frac{1}{8}F_5 \wedge *F_5 + \frac{1}{8\mathrm{i}}\frac{C_4 \wedge G_{(3)} \wedge \overline{G_{(3)}}}{\mathrm{Im}\tau}\right)\,. \tag{1.11}$$

Exercise: The action is invariant under SL(2,$\mathbb{Z}$):

$$\tau \mapsto \frac{a\tau + b}{c\tau + d}\,, \quad G_{(3)} \mapsto \frac{G_{(3)}}{c\tau + d}\,, \quad ad - bc \neq 0\,. \tag{1.12}$$

In particular, one specific element of the SL(2,$\mathbb{Z}$) is of the form

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,, \tag{1.13}$$

which transforms $\tau \mapsto -1/\tau$ and therefore $g_s \mapsto 1/g_s = g_s'$. Hence, one has a duality between a theory that is weakly coupled and a theory that is strongly coupled (weak-strong duality).

Note: How does the τ parameter relate two the shape of the two-torus? If one varies τ, one changes the shape and structure of the torus. F-theory is on very different footing as M-theory.

1.2.2 Remarks on D-branes and orientifolds

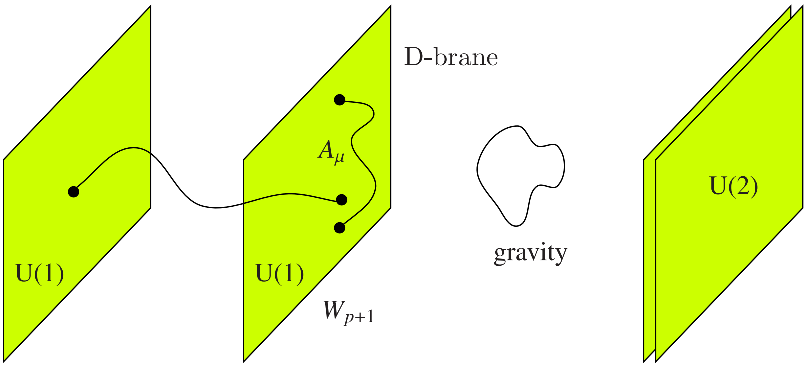
D-branes are objects, on which open strings can end, which leads to non-Abelian gauge symmetries.

Basis remarks on D-branes

The classical equations of motion of an open string admit two types of boundary conditions, namely Neumann (N) and Dirichlet (D) boundary conditions. With Dirichlet boundary conditions the endpoints of open strings are fixed to hypersurfaces, which are called D-branes. Under T-duality these two boundary conditions are exchanged. This is a very useful tool to check and motivate facts about D-branes. The D-branes come with **Chan-Paton charges**, which is information added to the endpoints of the string. The oriented open strings on a stack of N D-branes come with N^2 massless vector states, which form a U(N) gauge group. For **unoriented strings** (for example a Moebius stripe) one can have different gauge groups Sp(N) or SO(N).

1.2.3 D-branes in Type II string theory

The basic setup is that we have a number of D-branes in the ten-dimensional space (or some subspace). On D-branes there are open strings that give us the gauge fields A_μ.



The length of a string is proportional to its mass. To get a massless string, the distance between these two D-branes must go to zero. The D-branes break Lorentz invariance, because the specify a lower-dimensional subspace, if $p < 9$. This also implies that they also break supersymmetry. In Type II theories they can maximally preserve 16 out of the 32 SUSY charges. If branes preserve exactly half of the supercharges these are stable DPs branes. That such a breaking has to occur one can also look at the SUSY vector multiplets. Now to the Dp-brance charges. Recall the Maxwell equations with source:

$$\mathrm{d}F = *J_m\,, \quad d * F = J_e \Rightarrow e = \int_{S^2} *F\,, \quad g = \int_{S^2} F\,. \tag{1.14}$$

$$S_{\text{int}} = e\int A = e\int \mathrm{d}\tau\, A_\mu \frac{\partial x^\mu}{\partial \tau}\,. \tag{1.15}$$

The generalization would be

$$S_{\text{int}} = \mu_p \int_{W_{p+1}} \mathrm{i} * C_{p+1}\,, \tag{1.16}$$

whereas W_{p+1} is the world volume of the D-brane and C_{p+1} are the (RR)-forms of Type II string theory. C_{p+1} would be a type of Chern-Simons coupling. In other words: The brane is electrically charged under the (RR) forms and

$$q = \mu_p \int^{S^{D-p-2}} *F_{p+2}\,, \tag{1.17}$$

gives the charge of the brane at infinity. In Type IIA we had the odd forms C_1, C_3, C_5, C_7, and C_9, whereas C_1 and C_3 are dual to C_5 and C_7, respectively. In a Type IIB theory we had the even forms C_0, C_2, C_4, C_6, and C_8, whereas C_0 is dual to C_8 and C_2 is dual to C_6. C_4 is self-dual. The D3-branes are special, since C_4 is self-dual; electrical and magnetic charge is the same. SL$(2,\mathbb{Z})$ not only transforms the parameter τ, but also the forms B_2 and C_2:

$$\begin{pmatrix} B_2' \\ C_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} B_2 \\ C_2 \end{pmatrix}\,. \tag{1.18}$$

If one exchanges the C_2 with the B_2, something must happen, since the fundamental string is charged unter B_2 and under C_2 the D1 string. Hence, the duality interchanges D1 branes with fundamental strings. In general, we must be able to introduce a new object, that is charged with respect to B_2 and C_2, there must be something like (p, q)-strings. The same argument can be done with C_0 and C_8. There must exist (p, q)-7-branes. These connections are captured in the geometric framework of F-theory.

An important feature is the existence of stable D-branes that preserve half of the supercharges ("half BPS"). The idea is that the universe is on a D_p-brane.

1.2.4 D-brane world-volume actions

i embeds the $p+1$-dimensional world-volume W_{p+1} in $\mathcal{M}_{10}$. The effective action measures the volume of the brane. **Dirac-Born-Infeld action**:

$$S_{\text{DBI}}^{\text{string}} = -T_p \int\limits_{W_{p+1}} \mathrm{d}^{p+1}\xi \, \exp(-\phi)\sqrt{\det(i*(g+B_2)_{ab} + 2\pi\alpha' F_{ab})} \,. \tag{1.19}$$

D-branes are non-perturbative objects in the string coupling. An important tool is T-duality in order to perform the checks for the action. One starts with the action for the D9-brane. Then, one T-dualizes and goes down to the D8-, D7-brane etc. If one expands the determinant unter the square root with respect to F_{ab} , one finds at lowest order $\int \exp(-\phi) F \wedge *F$, which is the kinetic term of a gauge field in four-dimensions. There is another term, namely $\int \exp(-\phi)\sqrt{\det(g)}$, which gives the volume of the brane and multiplied by T_p its tension. The second part of the action is the **Chern-Simons action**; it includes the coupling to (RR) forms:

$$S_{\text{CS}} = \mu_p \int (i*(C^{\text{RR}})) \wedge \exp(i*B + 2\pi\alpha' F)\Big|_{p+1} \,, \tag{1.20}$$

where F is the U(1) field strength. C^{RR} is the sum over all (RR) forms of the theory:

$$C^{RR} = \sum_n C_n \,, \tag{1.21}$$

where n is odd for Type IIA and even for Type IIB theories. For a non-trivial F the brane can also be charged to the lower (RR) forms (for example for some kind of instanton configuration). In particular, there are **induced charges**

$$\mu_p \frac{(2\pi\alpha')^2}{2} \int i*(C_{p-3}) \wedge F \wedge F \,. \tag{1.22}$$

The induced D_{p-2} charge is

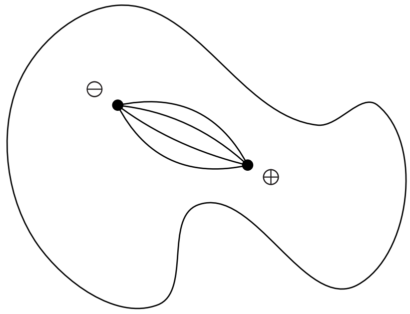
$$\mu_p \frac{(2\pi\alpha')^2}{2} = \frac{\mu_{p-4}}{8\pi^2} \,. \tag{1.23}$$

Remarks

- There have to exist non-Abelian generations of DBI and CS action.
- Non-Abelian D-branes can be charged with respect to higher RR-forms. A number of D3-branes can den dissolve into a D5 brane (Mayers).
- In general, there will be curvature couplings in a non-trivial brackground metric. The D-brane will "feel", if the space is strongly curved. There will be corrections to the action which go with the Riemann scalar.

1.2.5 Orientifold planes

The problem is easily clarified on a point particle picture. Consider a compact space and some particles in it. The field lines of a positively charged particle have to end on a negatively charged one. Something similar we want to have for D-branes with the difference that we not only want to cancel electromagnetic charges (RR-charges), but also gravitational charges (NSNS-charges).



This can be done using orientifold planes. These arise if one divides out (gauge fixes) a $\mathbb{Z}_2$ symmetry of a Type II background which contains Ω_p (world sheet parity reversal). Then also non-orientable world-sheets are allowed, as for example Klein's bottle. The projection will look like $\mathcal{O} = \Omega_p \mathcal{O}_0$, which leads to unoriented strings.

	ϕ	B_2	g	RR
Ω_p	+	-	+	IIA: $C_q \mapsto (-1)^{\frac{q-1}{2}} C_q$, q odd
				IIB: $C_q \mapsto (-1)^{\frac{q+2}{2}} C_q$, q even

The orientifold planes are fix-points of this involution $\mathcal{O}$ ($\mathcal{O}^2 = 1$). Candidates for $\mathcal{O}_0$ are

- $(-1)^{F_L}$: acts on NS-NS with $+1$ and on R-R with -1 (with F_L being the left-moving fermion number)
- geometric symmetry of the background: involution σ: $\mathcal{M}_{10} \mapsto \mathcal{M}_{10}$, $\sigma^2 = 1$

$$\mathbb{Z}_2 \begin{cases} \text{Type IIB: preserve orientation of } \mathcal{M}_{10} \\ \text{Type IIA: reverse orientation of } \mathcal{M}_{10} \end{cases} . \tag{1.24}$$

It is clear that the O-planes are the fix-points of the geometric involution σ: $\sigma(p) = p$.

Important examples

- In Type IIB theory: $\mathcal{O}_{\text{IIB}} = (-1)^{F_L} \Omega_p \sigma$

  For $\mathcal{O}$(field) = field the transformation properties σ^*: $\phi = \phi$, $B_2 = -B_2$, and $g = g$ have to be fulfilled. Check $\sigma^* C_q = (-1)^{q/2} C_q$.

- In Type IIA theory: $\mathcal{O}_{\text{IIA}} = (-1)^{F_L} \Omega_p \sigma$

  The properties for ϕ, B_2, and g remain the same, but $\sigma^* C_q = (-1)^{(q+1)/2} C_q$.

We want to come back to the orientifold planes, which are the σ-fix-points. There is no gauge theory on the orientifold planes. Hence, F is absent, so is B_2. (Check $i_{\text{O-plane}} * B_2 = 0$.) The orientifold action is then

$$S_{\text{OF}}^{\text{CS}} = \widetilde{\mu}_p \int i * C^{RR} , \quad S_{\text{OF}}^{\text{DBI}} = \widetilde{T}_p \int \exp(-\phi) \sqrt{\det(i * g)} , \tag{1.25}$$

$$\widetilde{T}_p = \mp 2^{p-s} T_p , \quad \widetilde{M}_p = \pm 2^{p+s} M_p . \tag{1.26}$$

1.3 Compactification to $D-4$

1.3.1 Consider bosonic Kaluza-Klein Reduction

The simple example due to Kaluza and Klein ist just a reduction from $D = 5$ to $D = 4$ with background $M_4 \times S^1$ (with coordinates $x^\mu \in M_4$ and $y \in S^1$). The reparameterization of the metric, which matches the *Ansatz* is given by:

$$g_{MN} = X^{-\frac{1}{3}} \begin{pmatrix} g_{\mu\nu} + X A_\mu A_\nu & X A_\mu \\ X A_\mu & X \end{pmatrix} . \tag{1.27}$$

For such a matrix one can always easily compute the determinant.

$$\langle g_{\mu\nu} \rangle = \eta_{\mu\nu} , \quad \langle X \rangle = 1 , \quad \langle A_\mu \rangle = 0 . \tag{1.28}$$

We perform the Kaluza-Klein reduction we expand with respect to the modes of the compact space S^1:

$$g_{\mu\nu} = \sum_n g_{\mu\nu}^{(n)}(x) \exp\left(\frac{2\pi i n y}{L}\right) . \tag{1.29}$$

The same has to be done for X and A_μ yielding the coefficients $X^{(n)}(x)$ and $A_\mu^{(n)}(x)$. Note that

$$\Delta_5 X = \Delta_4 X + \partial_y \partial_y X = \sum_n \left\{ \Delta_4 - \left(\frac{2\pi n}{L}\right)^2 \right\} X^{(n)} . \tag{1.30}$$

In four dimensions the higher modes with $n \neq 0$ have a mass $\sim 1/L$. The smaller the space S^1 the larger is the corresponding Kaluza-Klein mass. **At low energies** we only keep the massless modes $X^{(0)}$, $g_{\mu\nu}^{(0)}$ and $A_\mu^{(0)}$. Under this Kaluza-Klein reduction we obtain a four-dimensional effective action (with a prefactor that has to be rescaled by Weyl rescaling):

$$\int \frac{1}{2} *_5 R \Rightarrow S^{(4)} = -L \int \left(\frac{1}{2} *_4 R + \frac{1}{4} X^{(0)} F^{(0)} \wedge *F^{(0)} + \frac{(X^{(0)})^{-2}}{12} \mathrm{d}X^{(0)} \wedge *\mathrm{d}X^{(0)} \right) . \tag{1.31}$$

The relevant example is one that goes from $D = 10$ to $D = 4$. Under the specific split of the background $\mathcal{M}_4 \times \mathcal{M}_6$ the Lorentz group splits: $\mathrm{SO}(1,9) \mapsto \mathrm{SO}(1,3) \times \mathrm{SO}(6)$, whereas $\mathrm{SO}(1,3)$ is the local Lorentz group and $\mathrm{SO}(6)$ the holonomy group of $\mathcal{M}_6$. Later on, if we also take SUSY into consideration, $\mathrm{SO}(6)$ will be reduced to $\mathrm{SU}(3)$. The canonical *Ansatz* for the background metric is

$$\mathrm{d}s^{10} = g_{\mu\nu}^{(4)}(x)\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu + g_{mn}^{(6)}(y)\, \mathrm{d}y^m\, \mathrm{d}y^n . \tag{1.32}$$

In principle the *Ansatz* $g_{\mu\nu}^{(4)} \mapsto f(y) g_{\mu\nu}^{(4)}$ would still be Lorentz invariant, but we would have to deal with the additional factor $f(y)$ (warp factor), which is important in flux compactifications. Only if we additionally impose SUSY it will be insured that we can study the variations of the metric $g_{mn}^{(6)}$. Form fields:

$$\triangle A_p = 0 \Leftrightarrow d * F_{p+1} = 0 , \quad \triangle = d^* d + d d^* , \quad d^* = \pm * d * . \tag{1.33}$$

We now start to introduce forms both in four and in six dimensions:

$$(\triangle_4 + \triangle_6) A_p = 0 , \quad A_p = \sum_i \alpha_{p-i} \wedge \beta_i , \tag{1.34}$$

whereas $\alpha_{p-i}(x)$ is a $(p-1)$-form on $\mathcal{M}_4$ and $\beta_i(y)$ is a i-form on $\mathcal{M}_6$. This implies that $\Delta_6 \beta$ appears as a mass term in $D = 4$ for α. From this follows $\triangle_6 \beta_i = 0$, which are the harmonic i-forms on $\mathcal{M}_6$. Solving this equation is problematic for complicated spaces. However, we can use the **Hodge-theorem**, which tells us that every form can be decomposed into an harmonic, an exact and an diexact part. Hence, for every harmonic form β_i there is a cohomology class $[\beta_i]$.

$$H^p(\mathcal{M}_6, \mathbb{R}) = \frac{\text{closed } p\text{-forms}}{\text{exact } p\text{-forms}} = \frac{\{\mathrm{d}A_p = 0\}}{\{A_p = dB_{p-1}\}} , \tag{1.35}$$

which is reasonable, because in this definition no metric appears. Cohomology can therefore be defined as something topological.

1.3.2 Supersymmetry and Calabi-Yau manifolds $\mathcal{M}_6 = Y$

The goal is to study $\mathcal{M}_6$. When does $\mathcal{M}_6$ preserve some SUSY in $D = 4$? Our approach is the following:

i.) Since SUSY includes Lorentz invariance, in $D = 4$ $\langle A_\mu(x) \rangle = 0$ and $\langle \text{fermions} \rangle = 0$ in the background. Otherwise, the Lorentz group would be broken. Scalars however, can admit a vacuum expectation value, since they do not transform under the Lorentz group.

ii.) Check the SUSY variations: In particular, one has to check that $\langle \delta_{\mathrm{SUSY}} \text{bosons} \rangle = 0 = \langle \text{fermions} \rangle$, which is no additional condition to satisfy. However, the story is different for the SUSY variation of fermions:

$$\boxed{\langle \delta_{\mathrm{SUSY}} \text{fermions} \rangle = \langle \text{bosons} \rangle \overset{!}{=} 0 .} \tag{1.36}$$

In Type II supergravity we know the SUSY variations explicitly. In other words, the SUSY variations roughly look like this:

$$\delta\psi_M = \nabla_M \varepsilon + (F_p, H_3)\varepsilon\,, \tag{1.37}$$

whereas (F_p, H_3) are the field strength appearing in the bosonic action. Since $\langle F_p\rangle = \langle H_3\rangle = 0$ we have to assure that $\nabla_M\varepsilon = 0$. So, there has to be a spinor η on the internal manifold $\mathcal{M}_6$, which has to be covariantly conserved: $\nabla_m\eta = 0$. This implies that $\mathcal{M}_6$ is a Calabi-Yau manifold. Let us consider a Calabi-Yau n-fold, which is a complex n-dimensional manifold: $\dim_{\mathbb{C}}(Y_n) = n$. Y_n is compact and Kähler with $c_1 = 1/(2\pi)[R] = 0$, whereas R is the Ricci two-form. Theorem (Calabi-Yau): There exists a Kähler metric with SU(n) holonomy, in other words, a Ricci-flat metric. This Ricci-flat metric is not known analytically, but since in the Kaluza-Klein reduction the metric does not appear at any point, we can nevertheless perform computations despite this fact! On Calabi-Yau n-folds:

i.) there exists a Kähler form $J_{mn} = J_n{}^p g_{mp}$ with $J = \mathrm{i} g_{a\bar{b}}\,\mathrm{d}z^a \wedge \mathrm{d}\bar{z}^b$ and $\mathrm{d}J = 0$.

$$\mathrm{Vol}(\mathcal{M}_6) = \frac{1}{3!}\int J\wedge J\wedge J = \frac{1}{6}\int \sqrt{g}\,\mathrm{d}^6x\,. \tag{1.38}$$

ii.) There exists nowhere a vanishing holomorphic n-form Ω:

$$\Omega = \frac{1}{n!}\Omega_{\mathrm{abc}}(z)\,\mathrm{d}z^a\wedge\mathrm{d}z^b\wedge\mathrm{d}z^c\wedge\ldots\,,\qquad \boxed{\mathrm{d}\Omega = 0\,.} \tag{1.39}$$

Instead of working with the metric, which we do not know, we will work with the above two forms and study the perturbations. We do a Hodge decomposition of the cohomology group:

$$H^p(M,\mathbb{C}) = \sum_{k=0}^{p} H^{(k,p-k)}\,. \tag{1.40}$$

$H^{(k,p-k)}$ has k $\mathrm{d}z$-terms and $p-k$ $\mathrm{d}\bar{z}$-terms ($\mathrm{d}z\wedge\ldots\mathrm{d}\bar{z}\wedge\ldots$). The dimension of $H^{(k,p-k)}$ will be denoted as $h^{(k,p-k)}$ and the dimension of $H^p(M,\mathbb{C})$ are the Betty-numbers b^p. Euler characteristic:

$$\chi = \sum_p (-1)^p b^p\,. \tag{1.41}$$

$$\begin{array}{ccccccc} & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{21} & & 1 \end{array}$$

The Euler characteristic is then $\chi = 2(h^{1,1} - h^{2,1})$, whereas $h^{1,1}$ and $h^{2,1}$ are the only independent Hodge-numbers that characterize the Calabi-Yau manifold. These will give us exactly the allowed deformations of the two above forms. Now we can do the same for Calabi-Yau 4-folds. This is needed to reduce the 12-dimensional F-theory to four dimensions. The Hodge diamond then looks in the following fashion: How it appears that we have four independent Hodge numbers, but there is one relation, with which can be checked that $h^{2,2}$ is related to the other numbers:

$$h^{2,2} = 2(22 + 2h^{1,1} + 2h^{1,3} - h^{1,2})\,. \tag{1.42}$$

The Euler characteristic in this case is then

$$\chi = 6(8 + h^{1,1} + h^{3,1} - h^{2,1})\,. \tag{1.43}$$

So, there are three independent Hodge numbers: $h^{1,1}$, $h^{3,1}$, and $h^{2,1}$. $h^{1,1}$ will parameterize the deformations of the Kähler form and $h^{3,1}$ will parameterize the deformations of Ω. $h^{2,1}$ will give us additional scalars in the four-dimensional theory after compactification. Now we want to study metric deformations by performing linear deformations of the Calabi-Yau manifold: $g^0_{mn} \mapsto g^0_{mn} + \delta g_{mn}$. Since we want to go from one Calabi-Yau manifold to another one, we have to impose $R_{pq}(g^0_{mn}) = 0$ and $R_{pq}(g^0_{mn} + \delta g_{mn}) = 0$. These conditions will give us a differential equation for δg_{mn}. The solutions of these equations split into two sets, which correspond to deformations of the Kähler form and to deformations of Ω.

1.) $\delta g_{a\bar{b}} = -\mathrm{i}v^A(\omega_A)_{a\bar{b}}$ with the harmonic (1,1)-forms ω_A. The index structure is a (1,1) form and therefore this are Kähler structure deformations.

2.) For the second deformations both indices are complex ones:

$$\delta g_{ab} = \frac{\mathrm{i}}{\|\Omega\|^2}\bar{z}^h(\overline{\chi}_k)_{a\bar{c}\bar{d}}\Omega^{\bar{c}\bar{d}}{}_b\,, \quad \|\Omega\|^2 = \frac{1}{3!}\Omega_{mnp}\overline{\Omega}^{mnp}\,. \tag{1.44}$$

Harmonic (1,2)-forms $(\overline{\chi}_k)_{a\bar{c}\bar{d}}$ parameterize these deformations. Under mirror symmetry the two spaces are exchanged and this makes only sense, if both are complex spaces.

Effective action:

$$-\int\limits_{M_4\times Y_3}\frac{1}{2}R_{10}*_{10}1+\frac{1}{4}H_3\wedge H_3 \supset -\int\frac{1}{2}R_4*_4 1 + G_{k\bar{l}}(z,\bar{z})\,\mathrm{d}z^k\wedge *\mathrm{d}\bar{z}^l + C_{5,AB}(t,\bar{t})\,\mathrm{d}t^A\wedge *\mathrm{d}t^B\,, \tag{1.45a}$$

$$\boxed{G_{k\bar{l}} = -\frac{\int\limits_Y \chi_k\wedge\overline{\chi}_{\bar{l}}}{\int\limits_Y \Omega\wedge\overline{\Omega}}\,, \quad C_{5,AB} = \frac{1}{4\mathrm{Vol}(Y_3)}\int\limits_Y \omega_A\wedge *_6\omega_B\,.} \tag{1.45b}$$

The deformations in the four-dimensional effective theory depend on the four-dimensional coordinates $z^k(x)$ and $t^A(x)$. They are complex scalar fields in $D=4$. One can check that these two matrices are in fact Kähler matrices.

1.) $K^{\mathrm{CS}} = -\log\left(-\int\Omega\wedge\overline{\Omega}\right)$

2.) $K^{\mathrm{KS}} = -\log\left(\frac{1}{6}\int J\wedge J\wedge J\right)$

$$J = \frac{(t-\bar{t})^A}{2\mathrm{i}}\omega_A = v^A\omega_A\,. \tag{1.46}$$

The first one is a very complicated Kähler potential, since $\Omega(z)$ is a complicated function of a complex structure deformation z. (A_I, B^J) is a symplectic basis of three-cycles $H_3(Y_3,\mathbb{Z})$.

$$X^I(z) = \int\limits_{A^I}\Omega\,, \quad F_J(z) = \int\limits_{B_I}\Omega\,, \tag{1.47}$$

are the periods of Ω. Both quantities are determined explicitly by differential equations, namely the Picard-Fuchs equations. One can show that $F_J(z)$ is a derivative of a holomorphic function $F(z)$, which is known as the pre-potential: $\partial_{X^I}F(z)$. z^k and t^A have no potential, which is known as the moduli problem.

Chapter 2

Fluxes in String compactifications

2.1 Fluxes and no-go results

There are the moduli fields z^k, t^A, the dilaton ϕ; via the Kaluza-Klein reduction there are also the scalar zero-modes of the (RR)-forms. These are unwanted scalars in the four-dimensional spectrum and one wants to get rid of them. Deformations at Wilson lines of D-branes.

Dine-Seeberg: If SUSY is broken the scalar fields will not necessarily be protected against quantum corrections. These might generate a potential for these fields. Can we compute this potential? The argument is rather simple. Let us look at one single field t that parameterizes some couplings in the gauge theory. We assume that as $t \mapsto \infty$ we are at weak coupling, where we can trust our approximations. At this decoupling limit all volumes of the Calabi-Yau manifolds are large; however, this is necessary for supergravity approximation. Otherwise one would have to take into account all Kaluza-Klein modes etc. If quantum corrections generate a potential, it has to hold $\lim_{t\mapsto\infty} V(t) = 0$, because there was classically no potential for t. Hence, we would live in a non-perturbative vacuum und calculations would be very hard to be performed. Therefore, the minima have to be pushed in the direction of large t. An easier way would be not to consider quantum correction, but a mechanism that generates a potential at the classical level. At this step we are coming to the so-called **background fluxes**. These are non-trivial background values of the field strengths in the supergravity theory, namely $\langle H_3 \rangle$ (NS-NS), $\langle F_p \rangle$ (R-R) or $\langle F_0 \rangle = m$ (Romans mass of IIA SUGRA). If these are switched one, Lorentz invariance would be broken in four dimensions. The only way to switch them on consistently would be on the internal manifold $\mathcal{M}_6$. For example, in Type IIB one could switch on $\langle H_3 \rangle$, $\langle F_p \rangle$, take a basis $H_3(Y_3, \mathbb{Z}) \supset \{A^I, B_I\}$ and we integrate these fluxes over the basis:

$$\frac{1}{2\pi\alpha'} \int\limits_{A^I} H_s = m^I \, , \quad \frac{1}{2\pi\alpha'} \int\limits_{B^I} H_3 = e_I \, . \tag{2.1}$$

m^I and e_I are integers by Dirac quantization. The "flux quanta" ($h^{1,1}$, $h^{2,1}$, m^I, e_I) specify the vacuum. From the equations of motion for H_3 and F_3 follows $\mathrm{d}H_3 = \mathrm{d}F_3 = 0$.

IIB	CY	H_3	F_3	F_5	F_1	$H^3(Y, \mathbb{Z})$
IIA	CY	$F_0 = m$	F_2	F_4	F_6	H_3

$H^3(Y, \mathbb{Z})$ are the flux quantum numbers. For calculating the flux potential we take the ten-dimensional action, put it on the brackground and switch on the fields.

$$S_{\text{flux}} = \int\limits_{\mathcal{M}_4} *_4 1 \underbrace{\frac{m_p^4}{\text{Vol}(Y_3)} \int\limits_{Y_3} F_p \wedge *F_p}_{V_{\text{flux}}} \, . \tag{2.2}$$

Normalize (check how the quantities scale with overall value):

$$g_{mn}^{(6)} = r^2 g_{mn}^{(6)} \, , \quad \int\limits_{Y_3} *_6^0 1 = 1 \, . \tag{2.3}$$

$$\frac{V_{\text{flux}}}{m_p^4} = r^{-6-2p} \int\limits_{Y_3} F_p \wedge *_0 F_p \geq 0 \, . \tag{2.4}$$

We see that it runs away to $r \mapsto \infty$. To avoid this behavior, can we set

$$\int F_p \wedge *_0 F_p = 0\,? \tag{2.5}$$

This would lead to $F_p = 0$. So, we have generated a potential, but it is useless, since it is driven to a point, where the space compactifices. However, we really did something wrong: what we did not take into account was the backreaction on the geometry!

$$\triangle V = -r^{-8}\frac{1}{2}\int *_6^0 1 R_6^0\,. \tag{2.6}$$

One can check that there exist situations with AdS_d vacua.

$$M_{\mathrm{AdS}}^2 = \frac{V}{m_p^2}\,. \tag{2.7}$$

The problem is that the AdS scale is linked to the inverse size of the internal space and furthermore linked to the Kaluza-Klein scale. Hence, the vacua cannot be understood as small perturbations around Minkowski space. For small cosmological constant the included corrections are of $\mathcal{O}(M_{\mathrm{KK}})$ and therefore large (Freund-Rubin vacua).
There exists a no-go-theorem (Maldacena, Nunez) which says: For compact and smooth spaces $\mathcal{M}_6$, Minkowski or de Sitter space there are no fluxes other then F_1 and dual. Hence, we have to find a way around this theorem and this can be done by dropping the assumption about the “smoothness” of the space: we just take orientifolds! There exist solutions to the ten-dimensional equations of motion (also to the four-dimensional effective theory), which are Minkowski:

- $R_6 = 0$,
- 3-brane charges and tensions cancel,
- 7-brane charges and tensions cancel.

We have to check the consistency conditions. For that the Bianchi identity of $F_5 = dC_4 + 1/2 B_2 \wedge F_3 - 1/2 C_2 \wedge H_3$ have to be evaluated, whereas the 3-brane charges appear from local sources.

$$\mathrm{d}F_5 = H_3 \wedge F_3 + 2T_3 \zeta_3^{\mathrm{va}}\,, \qquad \boxed{\int H - 3 \wedge F_3 + 2T_3 Q_3^{\mathrm{lac}} = 0\,.} \tag{2.8}$$

This condition is called the “tadpole cancellation”. Contributions from fluxes and from sources have to cancel on compact space. Also the charges of D7/O7 have to cancel. Furthermore

$$\sum [D_{\mathrm{D7}}] = 4 \sum [D_{\mathrm{O7}}]\,, \tag{2.9}$$

evaluated on four-cycles Y_3 in $H_4(Y_3)$.

2.1.1 Effective $D = 4$ action

We want to consider $\mathcal{N} = 1$ in $D = 4$ supergravity.

$$S = -\int \frac{1}{2} * 1R + K_{i\bar{j}} DM^I \wedge *_4 \overline{DM^J} + \frac{1}{2}\mathrm{Re}(f_{\kappa\lambda} F^\kappa \wedge *F^\lambda) + \frac{1}{2}\mathrm{Im}(f_{\kappa\lambda} F^\kappa \wedge F^\lambda) + *1V\,, \tag{2.10}$$

with

$$V = \exp(K)(K^{IJ} D_I W \overline{D_J W} - 3|W|^2) + \frac{1}{2}\mathrm{Re}(f^{\kappa\lambda}) D_\kappa D_\lambda\,. \tag{2.11}$$

Both $f_{\kappa\lambda}(M^I)$ and $W(M^I)$ are holomorphic. K is the Kähler potential on the field space spanned by $(M^I, \overline{M}^J)$: $K_{IJ} = \partial_{M^I}\partial_{\overline{M}^J} K$. Notice: Holomorphic isometry splits the homology group:

$$H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)}\,, \quad H_{(p,q)} = H^+_{(p,q)} \oplus H^-_{(p,q)}\,. \tag{2.12}$$

The (p, q) structure is preserved since σ is holomorphic.

2.2 Fluxes in Type IIB

The key idea is to include orientifold planes and D-branes. These are non-smooth internal spaces. Projection:

$$O = (-1)^{F_L}\Omega_p\sigma\,, \quad \sigma : Y_3 \mapsto Y_3\,, \quad \sigma(\mathcal{M}_4) = \mathcal{M}_4 \text{ pointwise}\,, \quad \sigma^2 = 1\,. \tag{2.13}$$

SUSY implies that σ is holomorphic and isometric: $\sigma^* J = j$, $\sigma^*\Omega = -\Omega$. We can use O3/O7 planes and D3/D7 branes.

$$\mu_p = \tau_p \int \text{BPS}\,. \tag{2.14}$$

Include fluxes H_3 and F_3 through 3-cycles of tension of O3/D3 and induced O7/D7 tension:

$$\begin{aligned}\frac{V(r,\phi)}{m_p^4} = \exp(4\phi)\Bigg\{ & r^{-12}\Big(\exp(-\phi)T_3(g^0) + \underbrace{\int\limits_{Y-3} F_3\wedge *F_3 + \int\limits_{Y_3} H_3\wedge *H_3 - \exp(-\phi)\int\limits_{Y_3} F_3\wedge *H_3\Big)}_{\text{flux part}} \\ & + \underbrace{\frac{1}{r^8}\left(\exp(-\phi)T_7(g^0) - \exp(-2\phi)\int R_0 *_0 1\right)}_{\text{geometry part}}\Bigg\}\,.\end{aligned} \tag{2.15}$$

2.2.1 Chiral coordinates/scalars M^I

With z^k being the complex structure deformation induce the following form on Y_3:

$$\Pi^{\text{ev}} = \mathrm{i}\mathrm{Re}\widetilde{\Pi}^{\text{ev}} + \exp(-B_2)\sum_{p=0,2,4} C_p\,, \quad \widetilde{\Pi}^{\text{ev}} = \exp(-\phi)\exp(-B_2 + \mathrm{i}J)\,. \tag{2.16}$$

The NS-sector complexifies the R-sector in chiral multiplets.

$$D^\alpha \in H^+_{(2,2)}(Y_3) : \; T_\alpha = \int\limits_{D^\alpha} \Pi^{\text{ev}} = \mathrm{i}\int\limits_{D^\alpha} \exp(-\phi)J\wedge J + \ldots + \int\limits_{D^\alpha} C_4 + \ldots\,, \tag{2.17}$$

$$\Sigma_a \in H^-_{(1,1)}(Y_3) : \; G^a = \int\limits_{\Sigma_a} \Pi^{\text{ev}} = \int\limits_{\Sigma_a} C_2 - \tau\int\limits_{\Sigma_a} B_2\,, \tag{2.18}$$

$$\tau = \int\limits_{O} \Pi^{\text{ev}} = C_0 + \mathrm{i}\exp(-\phi)\,. \tag{2.19}$$

Kählerpotential:

$$\begin{aligned} K &= -\log\left(\mathrm{i}\int \Omega\wedge\overline{\Omega}\right) - 2\log\left(\widetilde{\Pi}^{\text{ev}} \text{“}\wedge\text{''}\, \overline{\widetilde{\Pi}^{\text{ev}}}\right) = \\ &= -\log\left(\mathrm{i}\int \Omega\wedge\overline{\Omega}\right) + \left(\exp(-2\phi)\int\limits_{Y_3} J\wedge J\wedge J\right) = \\ &= -\log\left(\mathrm{i}\int \Omega\wedge\overline{\Omega}\right) - \log(\tau - \overline{\tau}) - 2\log(\text{Vol}^E(Y_3))\,. \end{aligned} \tag{2.20}$$

Exercise: Check that the last two expressions are the same! The superpotential is given by:

$$W(z,\tau) = \int \Omega\wedge G_3\,, \quad G_3 = F_3 - \tau H_3\,, \tag{2.21}$$

whereas F_3 is the background flux. The procedure allows us to stabilize half of moduli (dilaton) of space (at small values such that the approximation is correct, otherwise there would be string scattering). Check:

$$V = \exp(K)(K^{i\bar{j}} D_i W \overline{D_j W})\,, \tag{2.22}$$

which does not stabilize the moduli T_α and G^a. The fact that it does not stabilize T_α is known as the KKLT scenario. This problem can be solved by including quantum corrections. Check for D_4 SUSY vacua. From $D_\pm W = 0$ follows

$$0 = D_{z^k} W = \int G_3 \wedge \chi_k \,, \quad 0 = D_T W = \frac{1}{\overline{\tau} - \tau} \int \overline{G}_3 \wedge \Omega \,, \quad 0 = D_{T_\alpha} W = K_{T_\alpha} W \Rightarrow W = 0 \,. \tag{2.23}$$

These together do not imply that the flux vanish, but that it is of a certain type: Hence, G_3 is not a (1,2)-form, not a (3,0)-form and not a (0,3)-form. G_3 is a (2,1)-form (flux landscape). This fact fixes τ and z^k. In fact, one can even show that one can get a fixing of a small string coupling.

2.3 KKLT scenario

In SUSY vacua V vanishes and one has Minkowski vacua. We want to stabilize the T_α, but for simplicity reasons we will just consider one of them: T ($h^{1,1}_+ = 1$, $h^{1,1}_- = 0$). Can we consider quantum corrections to these classical fluxes? There are two quantities for which one get quantum corrections:

i.) Kähler potential (LARGE volume scenarios)

ii.) superpotential (KKLT)

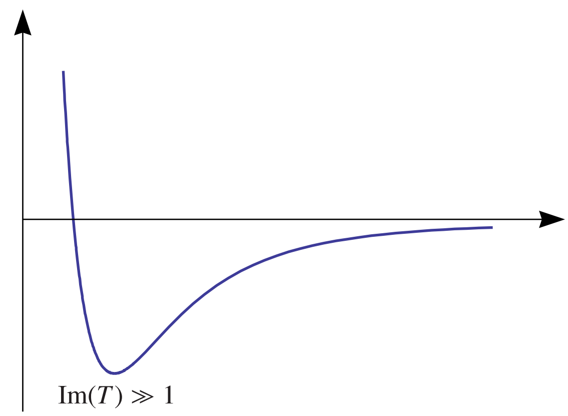
The first scenario breaks SUSY, whereas the second one does not. For $W(T)$ perturbative corrections are absent; the only way that corrections can appear is in a **non-perturbative** fashion. There can exist superpotentials which arise from D-brane instantons wrapped on cycles of the orientifold Y_3/σ. In this specific compactification these are the D3-brane instantons wrapped on four-cycles (Witten). The contribution of an instanton is of the form $W_{\mathrm{D3-superpot}} = A(z,\tau)\exp(\mathrm{i}aT)$, whereas $A(z,\tau) = A = \text{const.}$ in an effective theory. Here, we want to consider the KKLT mechanism:

1.) Fix z^k and τ at mass α'/r^3.

2.) The mass of T is much lower than the mass of z^k and τ. Hence, we can use an effective theory for T. What remains from the Kähler potential and from the superpotential is of the form

$$\boxed{K = -3\log(T - \overline{T})\,, \quad W = W_0 + A\exp(\mathrm{i}aT)\,.} \tag{2.24}$$

The W_0-part comes from the fluxes. For $W_0 \ll 1$ by **tuning** fluxes one gets the picture:



The minimum is pushed out to the large volume regime so that the approximation can be trusted. This implies a no-scale structure. The additional scale here is the exponentially small AdS mass at the minimum of the potential.

2.4 Fluxes in Type IIA orientifolds

In type IIA the orientifold projection is of the form $\mathcal{O}_{\mathrm{IIA}} = (-1)^{F_L}\Omega_p\sigma$. SUSY demands that this is an antiholomorphic involution:

$$\sigma^*\omega = \exp(2\mathrm{i}\theta)\overline{\Omega}\,, \quad \sigma^* J = -J\,. \tag{2.25}$$

This implies that the fixpoints are (4 − 3)-dimensional O6-planes or D6-branes. We can roughly evaluate the potential from all possible sources:

$$\frac{V}{m_p^4} = \exp(4\phi)\Big[\underbrace{N_R^2 r^{-14}}_{F_4} + \underbrace{N_{\text{NS}} r^{-12}\exp(-2\phi)}_{H_3} - \underbrace{N_{\text{O6}} r^{-9}\exp(-\phi)}_{\text{O6-planes}} + \underbrace{m^2 r^{-6}}_{\text{Romans mass}} \underbrace{- r^{-8}\exp(-2\phi)\int R^0}_{\text{Einstein-Hilbert}}\Big]\,. \tag{2.26}$$

- There exist AdS vacua with $R = 0$, $r \sim N_R^{1/4}$, $\exp(\phi) \sim N_R^{-3/4}$, $N_R \gg 1$, $N_{\text{NS}} \sim 1$, $N_{\text{O6}} \sim 1$, and $m \sim 1$. (By analyzing to consistency conditions, one finds that it is indeed possible to choose fluxes in this way.)
- The corresponding ten-dimensional solution has a smeared O6-plane.
- Apply **scalings**: $m_p^2 = \exp(-2\phi)r^6 \sim N_R^3$, $V/m_p^4 \sim -N_R^{-9/2}$

  $$M_{\text{AdS}}^2 = \frac{V}{m_p^2} \sim N_R^{-3/2} \ll M_{\text{KK}}\,. \tag{2.27}$$

  Despite this fact we are not in the examples that the AdS- and the KK-mass are connected like in the Freund-Rubin vacua.
- Consistency conditions:

  One can compute the tadpole cancellation conditions and this will leave us with the following equation, which looks very similar to the one found for the O7/D7:

  $$\sum [D_{\text{D6}}]_{\text{PD}} + mH_3 = 2[D_{\text{O6}}]_{\text{PD}}\,. \tag{2.28}$$

  H_3 ist the NS-NS-flux. Since $[D_{\text{O6}}]_{\text{PD}}$ is a small number, it is natural to have m small.

The next step is to evaluate the effective $D = 4$ action. For that we will perform a dimensional reduction on CY/σ. We split the cohomology: $H^p(Y_3) = H_+^p \oplus H_-^p$ and $H_p(Y_3) = H_p^+ \oplus H_p^-$. (The (p,q)-split is no longer possible, since it is an antiholomorphic evolution.) We choose chiral coordinates $J_c = B_2 + \mathrm{i}J$ and

$$t^a = \int_{\Sigma_a} J_c\,, \quad \Sigma_a \in H_2^-(Y_3)\,. \tag{2.29}$$

Furthermore, we will use

$$\Omega_c = \mathrm{i}\,\mathrm{Re}(C\Omega) + \mathrm{i}C_3\,, \quad C = \exp(-\phi - \mathrm{i}\theta)\left(\frac{\frac{1}{3!}\int J\wedge J\wedge J}{\mathrm{i}\int \Omega\wedge\overline{\Omega}}\right)^{\frac{1}{2}} = \exp(-\phi+\mathrm{i}\theta)\exp\left(\frac{K^{\text{CS}}}{2} - \frac{K^{\text{KS}}}{2}\right)\,. \tag{2.30}$$

We introduce the coordinates (A_κ, B^λ), which is a basis of three cycles H_3^+ of the Calabi-Yau manifold Y_3 with $\kappa = 1, \ldots, \widetilde{h}$, $\lambda = \widetilde{h} + 1, \ldots, b_+^3 = h^{2,1} + 1$ and $\dim H_3(Y_3) = (2h^{2,1} + 2)$. Furthermore

$$\boxed{N^\kappa = \int_{A_\kappa} \Omega_c\,, \quad T_\lambda = \int_{B^\lambda} \Omega_c\,,} \tag{2.31}$$

where $(t^a, N^\kappa, T_\lambda)$ are the chiral coordinates. The Kähler potential is:

$$K = -\log\left\{\frac{1}{3!}\int_{Y_3} J\wedge J\wedge J - 2\log\int_{Y_3} C\Omega\wedge\overline{C\Omega}\right\}\,. \tag{2.32}$$

Check:

i.) use linear multiplets ("dual formulation") and reduced/truncated $\mathcal{N} = 2$ special geometry

ii.) use work of Hitchin: take derivative of real part of K

Superpotential:

$$\boxed{W = \int\limits_{Y_3} \Omega_c \wedge H_3 + \int\limits_{Y_6} F_6 + \int\limits_{Y_3} J_c \wedge F_4 + \frac{1}{2!} \int\limits_{Y_3} J_c \wedge J_c \wedge F_2 + \frac{m}{3!} \int\limits_{Y_3} J_c \wedge J_c \times J_c \,.} \tag{2.33}$$

This superpotential depends on all moduli fields of the spectrum. It was shown that for $h^{2,1} = 0$, rigid Calabi-Yau Y_3 **all moduli** can be fixed by fluxes in a SUSY vacuum (de Wolfe, Kacher et al.). However, some problems remain:

- You have as many fluxes as moduli, hard time to tune value of moduli to wanted value (limited tunability).
- The O6/D6 backreact strongly on the geometry and deform the background. Normally one would say that there is a natural way to get rid of this problem by using M-theory on G_2 manifolds. However, no-one really knows how to deal with m in M-theory.
- Inflation, de Sitter etc is still under investigation.